Newton's laws

Force and Momentum

1 (a) 3.64×10^{-23} kg m s^{-1} (a small correction for relativity gives 3.68×10^{-23} kg m s^{-1})

(b) 6.97×10^{-24} kg m s^{-1}

(c) 480 kg m s^{-1}

2 $a = v^2/2s = 8.0\times10^{15}$ m s^{-2}, $F = ma = 7.29$ fN

3 $F = -0.981\text{ N}$, $a = -9.81\text{ m s}^{-2}$, $t = \sqrt{2s/a} = 3.06\text{ s}$

4 $a = -u^2/2s = -40.5\text{ m s}^{-2} = 4.13\ g$. $F = ma = 0.81\text{ MN}$

5 (a) $F = -\dfrac{dE}{dr} = -\dfrac{GMm}{r^2}$, directed radially inwards.

(b) $g = \dfrac{GM}{r^2} \Rightarrow M = \dfrac{gr^2}{G} = 6\times10^{24}\text{ kg}$

(c) 9.72 m s^{-2}.

6 (a) $F = -\dfrac{dV}{dr} = -k(r - r_e)$ (Hooke's law)

(b) Parabola (minimum at r_e), you can sketch a more realistic potential if you want

(c) Straight line with negative slope, passing through zero at r_e.

(e) Force in opposite direction to the deformation.

7 (a) Horizontal 346 m s^{-1}. Vertical 200 m s^{-1}.

(b) It will return to earth when in the vertical direction $s = 0 \Rightarrow v = -u \Rightarrow t = 2u/g = 40.8$ s

(c) Horizontal velocity is constant, hence $s = ut = 14.1$ km

8 Balancing forces, $mg = 6\pi\eta a v_t \Rightarrow v_t = \dfrac{mg}{6\pi\eta a}$

9 (a) a 150 g cricket ball moving at 40 m s^{-1}, 0.24 N

(b) a 13 g bullet moving at 700 m s^{-1}, 6.37 N

(c) a 1500 kg car moving at 200 km h^{-1}, 387 N

(d) a 1.8×10^{5} kg airliner moving at 2240 km h^{-1}? 69.7 MN

10 (a)
1. Every body remains at rest or in a state of uniform motion in a straight line unless acted upon by an external force. Molecule of gas between collisions.
2. Force is the rate of change of momentum.
3. To every action there is an equal and opposite reaction. Gas pressure.

(b) Potential energy function and one specification of position and momentum are sufficient.

(c) i. $\dfrac{dv_z}{dt} = -\dfrac{C}{m} v_z - g$

ii. The equation is integrated $\displaystyle\int_0^v \frac{m\,dv_z}{Cv_z + mg} = -\int_0^t dt$ to give $\left[\ln\left(\dfrac{C}{m} v_z + g\right)\right]_0^v = -\dfrac{Ct}{m}$,

exponentiating, $\dfrac{Cv}{m} + g = g\exp\left(-\dfrac{Ct}{m}\right)$ o r $v = \dfrac{mg}{C}\left(\exp\left(-\dfrac{Ct}{m}\right) - 1\right)$, this is negative because the velocity is directed downwards, and so is negative, the speed is the absolute value as required.

iii. Exponential growth to asymptotic limit $v_t = \dfrac{mg}{C}$

11 (a) Define the terms linear momentum and kinetic energy. How are the two quantities related? How is the change in kinetic energy related to the work done by a (conservative) force?

$\mathbf{p} = m\mathbf{v}$, $E = mv^2/2 = p^2/2m$. For a conservative force the change in KE is equal to the work done.

(b) Define the term potential energy. How is the change in potential energy related to the work done by a (conservative) force? In the light of your answer to (a), what is the significance of this result?

Minus integral of force relative to some location defined as the zero of the scale.
Change in potential energy is minus the work done.
For a conservative system kinetic + potential energy is conserved.

(c) The potential energy between two argon atoms varies with the interatomic separation, r, approximately according to the equation.

$$V(r) = 4\varepsilon\left(\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right)$$

with $\varepsilon = 1.7 \times 10^{-21}$ J and $\sigma = 3.4 \times 10^{-10}$ m.
i. Sketch the variation in potential energy as a function of argon atom separation.
ii. How does the force exerted on the argon atoms vary with atomic separation? At what separations is the force between the atoms attractive and at what separations is it repulsive?

$F = -\dfrac{dV}{dr} = \dfrac{4\varepsilon}{r}\left(12\left(\dfrac{\sigma}{r}\right)^{12} - 6\left(\dfrac{\sigma}{r}\right)^{6}\right)$. For attraction, $F > 0 \Rightarrow 2\left(\dfrac{\sigma}{r}\right)^{12} > \left(\dfrac{\sigma}{r}\right)^{6} \Rightarrow r < 2^{1/6}\sigma$

and for repulsion $F < 0 \Rightarrow r > 2^{1/6}\sigma$.